Overview

1. AAC Introduction and Theory
2. AAC Transfer Function Characterization
3. AAC Size Distribution Inversion and Validation
4. Effects of AAC Classifier Conditions
Equivalent Particle Diameters

\[ \tau = m \cdot B = \frac{C_c(d_a) \cdot \rho_0 \cdot d_a^2}{18\mu} = \frac{C_c(d_m) \cdot \rho_{\text{eff}} \cdot d_m^2}{18\mu} = \frac{C_c(d_{ve}) \cdot \rho_p \cdot d_{ve}^2}{18\mu \cdot \chi} \]

Where \( m \) is the particle mass, \( B \) is the particle mobility, \( C_c \) is the Cunningham Slip Correction, \( \mu \) is the viscosity of the surrounding gas, \( \rho_0 \) is unit density (1000 kg/m\(^3\)), \( \rho_{\text{eff}} \) is the effective density of the particles, \( \rho_p \) is the particle material density, \( d_{ve} \) is the volume equivalent diameter and \( \chi \) is the particle shape factor.

1. AAC Introduction and Theory
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Animation provided by Cambustion (http://www.cambustion.com/products/aac)

Aerodynamic Aerosol Classifier
AAC Transfer Function (TF) - Balanced Flows

- Non-diffusing (ND) transfer function is based on the particle streamline model (Tavakoli and Olfert, 2013)

- Diffusing (D) transfer function assumes that diffusion spreads the particles in a Gaussian distribution about the particle streamline model (Tavakoli and Olfert, 2013)

- Lognormal (Log) approximation of the AAC transfer function was calculated following the theory developed by Stolzenburg and McMurry (2008) to represent the DMA transfer function lognormally

AAC Setpoint: \( \tau^* = \frac{Q_{sh}+Q_{exh}}{\pi w^2 (r_1+r_2)^2 L} \)

Non-dimensional Flow Parameter: \( \beta = \frac{Q_a}{Q_{sh}} \)
TF Characterization using a Tandem AAC Setup

- Upstream AAC (AAC 1) is set at a constant setpoint and selects one aerodynamic particle diameter from the poly-dispersed aerosol source.
- Downstream AAC (AAC 2) steps through the aerodynamic diameter domain of the classified particles and records the corresponding doubly classified particle concentration at each setpoint.

2. AAC Transfer Function Characterization
Parameterized TF for Tandem AAC Deconvolution

- Similar to Martinson et al.’s (2001) characterization of the Differential Mobility Analyzer (DMA) transfer function, the AAC transfer function was parameterized to capture non-ideal behaviour, such as particle diffusion and losses, using:
  - Transmission Efficiency \( \lambda_\Omega \)
    Scales area under transfer function
  - Transfer Function Width Factor \( \mu_\Omega \)
    Scales transfer function FWHM

\[
\Omega_{NI}(\tau, \tau^*, \beta, \lambda_\Omega, \mu_\Omega) = \begin{cases} 
\lambda_\Omega \cdot \mu_\Omega \left[ 1 + \frac{\mu_\Omega}{\beta} \cdot \left( \frac{\tau}{\tau^*} - 1 \right) \right] & \text{if } \left(1 - \frac{\beta}{\mu_\Omega}\right) \cdot \tau^* \leq \tau \leq \tau^* \\
\lambda_\Omega \cdot \mu_\Omega \left[ 1 + \frac{\mu_\Omega}{\beta} \cdot \left( 1 - \frac{\tau}{\tau^*} \right) \right] & \text{if } \tau^* < \tau \leq \left(1 + \frac{\beta}{\mu_\Omega}\right) \cdot \tau^* \\
0 & \text{elsewhere}
\end{cases}
\]

AAC Setpoint: \( \tau^* = \frac{Q_{sh} + Q_{exh}}{\pi w^2 (r_1 + r_2)^2 L} \)

Non-dimensional Flow Parameter: \( \beta = \frac{Q_a}{Q_{sh}} \)
2. AAC Transfer Function Characterization
Transmission Efficiency, $\lambda_\Omega$

- AAC transmission efficiency ($\lambda_{\Omega,\text{AAC}}$) at aerodynamic diameter ($d_a$) can be estimated from:
  \[ \lambda_{\Omega,\text{AAC}} = \lambda_D (d_a) \cdot \lambda_e \]

- DMA transmission efficiency ($\lambda_{\Omega,\text{DMA}}$) at electrical mobility diameter ($d_m$) can be estimated from:
  \[ \lambda_{\Omega,\text{DMA}} = \lambda_D (d_m) \cdot \lambda_e \cdot f_n(d_m) \]

Where:
- $\lambda_e$ is the losses due to classifier entrance/exit effects
- $\lambda_D$ is the diffusional penetration (Karlsson et al., 2003):
  \[ \lambda_D = \begin{cases} 0.819e^{-11.5\delta} + 0.0975e^{-70.1\delta} + 0.0325e^{-179\delta} & \delta \geq 0.007 \\ 1 - 5.50\delta^{2/3} + 3.77\delta + 0.814\delta^{3/2} & \delta < 0.007 \end{cases} \]
- $f_n$ is the fraction of particles with mobility diameter $d_m$ neutralized to a minus one charge state [estimated by Wiedensohler (1988) and Gunn et al. (1956)].

- The non-dimensional deposition parameter ($\delta$):
  \[ \delta(d_p) = \frac{L_{\text{eff}} \cdot D(d_p)}{Q_a} \]

Where $L_{\text{eff}}$ is the length of a circular tube with the same diffusion deposition as the classifier, $D$ is the diffusion coefficient of the particles with diameter $d_p$ and $Q_a$ is the aerosol flowrate into the classifier.
The transfer function width factor of the AAC ($\mu_{\Omega,AAC}(d_a)$) or DMA ($\mu_{\Omega,DMA}(d_m)$) can be estimated from:

$$\mu_{\Omega}(d_p) = a \cdot d_p^b + c$$

Where $d_p$ is the particle diameter in nm.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMA$^a$</td>
<td>-11.05</td>
<td>-1.739</td>
<td>0.9956</td>
</tr>
<tr>
<td>AAC LF</td>
<td>-1.202</td>
<td>-0.2663</td>
<td>0.8805</td>
</tr>
<tr>
<td>AAC HF</td>
<td>7.144e-06</td>
<td>1.229</td>
<td>0.4947</td>
</tr>
</tbody>
</table>

$^a$ Based on data collected by Birmili et al. (1997)
This setup measures the aerodynamic size distribution \( \frac{dN}{d\log d_a} \) of a steady-state aerosol.

The AAC steps through the aerodynamic diameter domain of the aerosol source and records the corresponding classified particle concentration as a function of its aerodynamic diameter setpoint.
 AAC Inversion- Raw Measurements to dN/dlogd

• Stolzenburg and McMurry (2008) determined:

\[ N_i = \int \eta_i (d_a) \cdot \Omega (t_i) \cdot dN_i \]

Where \( N_i \) is the particle concentration downstream of the classifier, \( \eta_i \) is the particle detector counting efficiency and \( \Omega \) is the classifier transfer function at particle relaxation time setpoint \( t_i \).

• Applying the AAC Non-Ideal Transfer Function to this equation yields the following solution:

\[ \left. \frac{dN}{d\log d_a} \right|_{i, NI} = \frac{\ln(10) \cdot N_i}{\eta_i \cdot \left. \frac{d\log d_a}{d\log \tau} \right|_i \cdot \beta^*_i,NI} \]

Where \( \beta^*_i,NI \) is a non-dimensional parameter that describes the transfer function resolution, and incorporates the transmission efficiency factor \( (\lambda_\Omega) \) and width factor \( (\mu_\Omega) \) previously determined.
AAC Inversion Validation- AAC vs SMPS Theory

- To validate the AAC inversion, including its transfer function parameters ($\lambda_\Omega$ and $\mu_\Omega$), an AAC and SMPS were used in parallel to characterize the same aerosol source, however:
  - The SMPS measures the particle electrical mobility size spectral, $\frac{dN}{d\log(d_m)}$
  - The AAC measures the particle aerodynamic size spectral, $\frac{dN}{d\log(d_a)}$

Therefore, the AAC’s equivalent electrical mobility size distribution was calculated from its measured aerodynamic size distribution by:

$$\frac{dN}{d\log(d_m)} = \frac{dN}{d\log(d_a)} \cdot \frac{k \cdot d_m^{D_m-1}}{\rho_0 \cdot d_a} \cdot \frac{[D_m^{-1} + \frac{2.34 \cdot \lambda \cdot (d_m^{-2}) + 1.05 \cdot \lambda \cdot \exp(-0.39 \cdot \frac{d_m}{\lambda}) \cdot \left(\frac{d_m^{-2}}{d_a} - \frac{0.39}{\lambda}\right)]}{[2 \cdot d_a + 2.34 \cdot \lambda + 1.05 \cdot \lambda \cdot \exp(-0.39 \cdot \frac{d_a}{\lambda}) \cdot \left(1 - \frac{0.39 \cdot d_a}{\lambda}\right)]}$$

- Derived from the definition of particle relaxation time: $\tau = \frac{C_c(d_m) \cdot \rho_{eff} \cdot d_m^2}{18 \cdot \mu}$
- Assumes fractal effective particle density: $\rho_{eff}(d_m) = k \cdot d_m^{D_m-3}$
- Cunningham slip correction function was estimated following Allen and Raabe (1985)
AAC Inversion Validation- AAC vs SMPS Results

- DOS nebulized by constant output atomizer
- Both the SMPS multiple-charge correction $\alpha$, and AAC losses/broadening correction were significant and required
- High degree of agreement between corrected AAC and SMPS/CPC measurements ($CMD$, $GSD$ and $N_{total}$ agreement of -0.8%, 1.2% and 1.4% respectively)

<table>
<thead>
<tr>
<th></th>
<th>CMD (nm)</th>
<th>GSD</th>
<th>$N_{total}$ (p/cm$^3$)</th>
<th>Percent Difference from:</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMPS Raw Data</td>
<td>212.90</td>
<td>1.82</td>
<td>3.12E+04</td>
<td>62.6%</td>
</tr>
<tr>
<td>AAC LF Raw Data</td>
<td>258.11</td>
<td>1.94</td>
<td>1.37E+04</td>
<td>-28.5%</td>
</tr>
<tr>
<td>SMPS MC Corrected</td>
<td>245.58</td>
<td>1.98</td>
<td>2.15E+04</td>
<td>11.8%</td>
</tr>
<tr>
<td>AAC LF $\lambda$ and $\mu$ Corrected</td>
<td>243.58</td>
<td>2.00</td>
<td>1.95E+04</td>
<td>1.4%</td>
</tr>
<tr>
<td>CPC (Direct Measurement)</td>
<td>N/A</td>
<td>N/A</td>
<td>1.92E+04</td>
<td>N/A</td>
</tr>
</tbody>
</table>

*SMPS multiple-charge correction was applied following He et al. (2013) with the particle charging fractions estimated by Wiedensohler (1988) and Gunn et al. (1956).*
Other Considerations: Varying Classifier Conditions

- Decarlo et. al (2004) determined:

\[ d_a = d_{ve} \sqrt{\frac{1}{\chi} \cdot \frac{\rho_p}{\rho_o} \cdot \frac{Cc(d_{ve})}{Cc(d_a)}} \]

Where \( d_a \) is the particle aerodynamic diameter, \( d_{ve} \) is the particle volume equivalent diameter, \( \chi \) is the shape factor and \( Cc \) is the Cunningham slip correction.

- Since \( d_{ve} \) in an intrinsic particle property, it can be used to relate the change in \( d_a \) at different conditions (i.e. classifier versus reference):

\[
\frac{d_a,\text{class}}{d_a,\text{ref}} = \sqrt{\frac{Cc(d_{ve})_{@ \text{Classifier Conditions}}}{Cc(d_{ve})_{@ \text{Reference Conditions}}} \cdot \frac{Cc(d_a,\text{ref})_{@ \text{Reference Conditions}}}{Cc(d_a,\text{class})_{@ \text{Classifier Conditions}}} \}
\]

Assumes \( \chi \) is constant over regimes (\( x_c \approx \chi_t \approx x_v \))

4. Affects of Classifier Conditions
\textbf{Kn at Classifier versus Reference Conditions}

Where:

- $C_c$ is only a function of Knudsen Number ($Kn$)
- At the same conditions: $Kn(d_{ve}) = Kn(d_a) \frac{d_a}{d_{ve}} = Kn(d_a) \sqrt{\frac{1}{\chi} \cdot \frac{\rho_p}{\rho_o}} \cdot \frac{Cc(Kn(d_{ve}))}{Cc(Kn(d_a))}$
Considering Normal AAC Operating Conditions

4. Affects of Classifier Conditions
Summary

- The AAC is a novel instrument that classifies particles based on their aerodynamic diameter.

- A tandem AAC setup was used to characterized the transfer function of individual AACs and experimentally determined:
  - High transmission efficiencies ($\lambda_\Omega \approx 80\%$); and
  - Transfer function broadening higher than predicted by theory ($\mu_\Omega \approx 0.45$ to $0.75$).

- The AAC transfer function inversion theory was developed and validated experimentally as shown by the high degree of agreement with SMPS measurements completed in parallel.

- The change in the selected particle aerodynamic diameter due to varying classifier temperature and pressure is negligible ($<1\%$) within the AAC operating range.
References:


Questions?

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